Thinking The Unseen

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In *Thinking Without Words,* we discussed the question of the nature of certain types of thinking. I would like to follow this up with a bit of a longer exposition along similar lines, but with a different theme. Begin with some basic questions: We can state the ways we express subtle, but meaningful thoughts (poetry, painting, music, etc…) but are we aware of the form these thoughts take in the first place? Can you see yourself thinking? How do your thoughts present themselves to your subconscious, before they've reached the stage of being translated into something communicable? Looking to Einstein for answers, we found out that he didn't think in words, but in terms of music. This left us wondering if crucial scientific (and artistic) thinking is sensual in any way, as commonly accepted. Let's look at the most prized of the senses in terms of thinking, visual thinking.[1] Now, I'm not talking about the visual imagination we use when reading a book. What I would like to look at is, what Einstein is doing when he thinks visually about things that can't be seen. Think back to his thought- experiment which was a seed crystal to his special relativity: "First came such questions as: What if one were to run after a ray of light? If one were to run fast enough, would it no longer move at all?" What does an electro-magnetic wave look like? Has anyone seen light itself? What does light look like when it's standing still?

As summary of what we will discuss to approach this: We will look at how Nicholas of Cusa stretches the imagination beyond what it can visualize or reason, using the example of the circle in the infinite being the same as a line in the infinite. Then we'll look at the case of Gottfried Leibniz' demonstration of the calculus (an infinitely small point which still maintains a proportion.) This will lead into Bernhard Riemann's work, and into the same issue of the non-visualizable in Einstein's four-dimensional, curved space-time.

As we have seen in other places,[2] we can't even take for granted the objects we think we see; so, what's going on when we try to visualize non-objects? Great thinkers do not think in terms of objects,[3] or even in terms of relations between objects. There's the visual imagination for constructing models and navigating and such; but, when we're dealing with concepts like the atom, gravity, justice, etc., all perceptions fall infinitely short. These become important questions when you're trying to imagine the future of mankind—something which can't be seen because it doesn't physically exist yet.

Maximum Sight

Start with Cusa's *De Docta Ignorantia*[4] as an exercise, which begins with the following premise: "The more one

1. For a good counterposition to the argument I'm NOT trying to make, read the final chapter of Oliver Sacks's book, *The Mind's Eye,* and Gerald Holton's paper *The Art of the Scientific Imagination*

2. ΓΝΩΒΘΙ ΣΕΑΥΤΟΝ (Know Thyself), http://larouchepac.com/node/21346

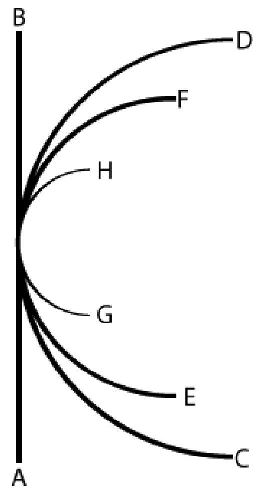
3. But there are thought-objects, what Riemann would call a 'Geistesmasse' or what the psychologist would call a 'Gestalt.' A close relative of Einstein tells us: "He works like an artist. First he sees the outlines, you may say the vision, of a great thought, and then he sets to work to substantiate it, to give it body and soul."

4. A good caution from Cusa: "When we set out to investigate the Maximum symbolically, we must leap beyond simple likeness. For since all mathematicals are finite and otherwise could not even be imagined: if we want to use finite things as a way for ascending to the unqualifiedly Maximum, we must first consider finite mathematical figures together with their characteristics and relations. Next, we must apply these relations, in a transformed way, to corresponding infinite mathematical figures.

knows that one is unknowing, the more learned one will be." Since every inquiry uses comparative relation, the infinite will always be unknowable since it escapes all comparative relation. As a consequence, the intellect never comprehends truth so precisely that it cannot comprehend it infinitely more precisely—we can always learn more. Cusa draws the analogy that the intellect is to truth "as an inscribed polygon is to the inscribing circle"—one always approaching, but never obtaining the other. Then he proceeds to talk about the maximum, because "we want to discuss the maximum learning of ignorance." In Book I, Ch13, he stretches the mind beyond the physical or the visual, by going into the infinite with figures, where you can only use the imagination to think of their possible characteristics.

> "So if the curved line becomes less curved in proportion to the increased circumference of the circle, then the circumference of the maximum circle, which cannot be greater, is minimally curved and therefore maximally straight."

To supplement the reader's understanding, Cusa illustrates the example with the following pedagogy:



> "…we can visually recognize that it is necessary for the maximum line to be maximally straight and minimally curved. Not even a scruple of doubt about this can remain when we see in the figure here at the side that arc CD of the larger circle is less curved than arc EF of the smaller circle, and that arc EF is less curved than arc GH of the still smaller circle. Hence, the straight line AB will be the arc of the maximum circle, which cannot be greater. And thus we see that a maximum, infinite line is, necessarily, the straightest; and to it no curvature is opposed. Indeed, in the maximum line curvature is straightness."

Your senses allow you to see a circle, reason allows you to follow the continuous process of the growing circle, but only a leap of the intellect, a stretching of the imagination, allows you to see the infinite circle as an infinite line.

> "In like manner, you can see that a triangle is a line. For any two sides of a quantitative triangle are, if conjoined, as much longer than the third side as the angle which they form is smaller than two right angles. For example, because the angle BAC is much smaller than two right angles, the lines BA and AC, if conjoined, are much longer than BC. Hence, the larger the angle, e.g. BDC, the less the lines BD and DC exceed the line BC, and the smaller is the surface. Therefore, if, by hypothesis, an angle could be two right angles, the triangle would be resolved into a simple line."
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> "Hence, by means of this hypothesis, which cannot hold true for quantitative things, you can be helped in ascending to non-quantitative things; that which is impossible for quantitative things, you see to be altogether necessary for non-quantitative things. Hereby it is evident that an infinite line is a maximum triangle. Q.E.D."

When we're dealing with functions of a higher quality than the usual geometric figures, we can use quantitative objects, by pushing them to their boundary of usefulness, to see what lies infinitely beyond their reach as a sort of negative foil.

Minimum Sight

Next, going from the maximum to the minimum, let's take a look at Leibniz's Calculus, his answer to Kepler's challenge[5] to bring physics into the hands of man. By way of introduction into the problem, read from Leibniz' dialog on continuity and motion:

> "Charinus: If I may be allowed to offer an inexpert opinion on such matters, I would declare that the transition from Geometry to Physics is difficult, and that we need a science of motion that would connect matter to forms and speculation to practice—something I learned from experiments of various kinds in my early military training. For I was often unsuccessful in trying out new machines and other delightful tricks of the trade, because the motions and forces involved could not be drawn and subjected to the imagination in the

5. New Astronomy Website on http://science.larouchepac.com/kepler/newastronomy Ch 60. This problem comes up with the elliptical orbits, something far from linear, which are always changing.

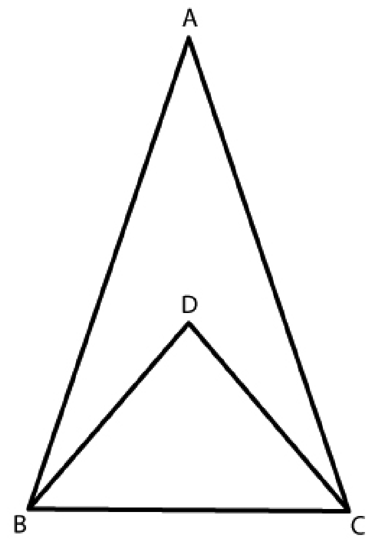
same way as figures and bodies could. For whenever I conceived in my soul the structure of a building or the form of a fortification, to begin with I would reinforce my wavering thought with tiny models made of wood or some other material. Afterwards, when I was more advanced, I was content to represent solids by plane drawings; and finally I gradually evolved such a facility of imagining that I could picture in my mind the whole thing complete with all its numbers, and could form vivid expressions of all its parts, and contemplate them as if they were in front of my eyes. But when it came to motion, all my care and diligence were of no use, and I could never reach the point where one might comprehend the reasons and causes of forces by the imagination, and form an opinion about the success of machines. For I always became stuck at the very beginning of an incipient motion, since I had noticed that what must come about in the whole of the remaining time must somehow already happen at the first moment. But to reason about moments and points, I had to admit, was indeed beyond my grasp. This is why, let down by my reasonings, I was reduced to relying on my own and other people's experience. But this experience often deceived us, as often as we had assumed false causes for the things we had experienced instead of the true ones, and had extended the arguments from them to things which to us seemed similar."

Modeling for engineering something is very useful, not to mention the countless other uses of imagery, yet still there's something above this that cannot be touched by the visual imagination, e.g. anything which involves a process (as in the unseen laws causing motion) or the ability to forecast outcomes of such processes.

It can't be taken up here in full, but Leibniz's development of the calculus (a philosophical discovery in its essence), using invisibles to deal with the tangible world, ruffled a few feathers. Leibniz, in a letter to Varignon, tries to explain the infinitesimal, using the law of continuity:

"… to make sure that there are lines in nature which are infinitely small in a rigorous sense, in contrast to our ordinary lines, [and] in order to avoid subtleties and to make my reasoning clear to everyone, it would suffice here to explain the infinite through the incomparable, that is, to think of quantities incomparably greater or smaller than ours… It is in this sense that a bit of magnetic matter which passes through glass is not comparable to a grain of sand, or this grain of sand to the terrestrial globe, or the globe to the firmament… it follows from our calculus that the error will be less than any possible assignable error, since it is in our power to make this incomparably small magnitude as small as we wish…It follows from this that even if someone refuses to admit infinite and infinitesimal lines in a rigorous metaphysical sense and as real things, he can still use them with confidence as ideal concepts which shorten his reasoning, similar to what we call imaginary roots in the ordinary algebra…"

Mathematicians of the day were uncomfortable with relative measures (as in, they were stuck in the belief in fixed measures) and were therefore afraid of using a quantity which they couldn't compare to their own rulers. The infinitesimal as an idea had to be fought for in very much the same way as the "numbers" such as zero, fractions, negative numbers, "irrationals," and "imaginaries" had, in order to gain their civil rights. Again, Leibniz used a clear analogy to demonstrate his idea, by taking something visibly simple and expanding it past the imaginatively visible. The demonstration goes as follows:

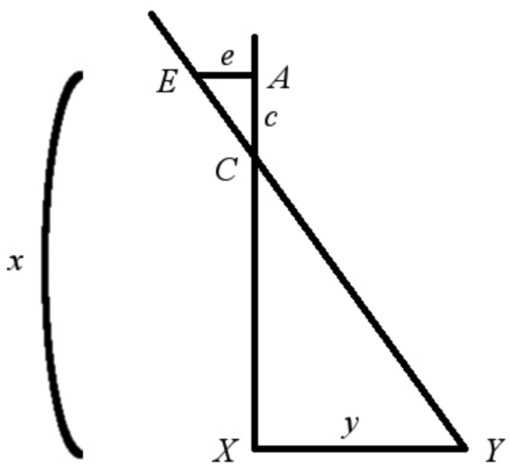


Since triangles *CAE* and *CXY* are similar, it follows that $(x-c)/y = c/e$. Consequently, if the straight line *EY* more and more approaches the point *A*, always preserving the same angle at the variable point *C*, the straight lines *c* and e will obviously diminish steadily, yet the ratio of *c* to e will remain constant. Here we assume that this ratio is other than 1 and that the given angle is other than 45 degrees.

Now assume the case when the straight line *EY* passes through *A* itself; it is obvious that the points *C* and *E* will fall on *A*, that the straight lines *AC* and *AE*, or *c* and e, will vanish, and that the proportion or equation *(x-*

> $c)/y = c/e$ will become $x/y = c/e$. Then in the present case, $x\text{-}c = x$. Yet c and e will not be absolutely nothing, since they still preserve the ratio of CX to XY. For if c and e were nothing in an absolute sense in this calculation, in the case when the points C, E, and A coincide, c and e would be equal, since one zero equals another, and the equation or proportion $x/y = c/e$ would become $x/y = 0/0 = 1$; that is, $x=y$, which is an absurdity, since we assumed that the angle is not 45 degrees. Hence c and e are not taken for zeros in this algebraic calculus, except comparatively in relation to x and y; but c and e still have an algebraic relation to each other. And so they are treated as infinitesimals, exactly as are the elements which our differential calculus recognizes in the ordinates of curves for momentary increments and decrements.

An infinitesimal triangle has infinitesimal sides which have different characteristics to each other in order to hold a ratio. In other words, just because you can't see these lines, doesn't make them simply zero, nor does it make them infinity, i.e. incomprehensible. This invention came at an early period of science when things like microscopic organisms were just coming into knowledge, and was still a ways from a working knowledge of molecules and atoms. It's hard to imagine what the world of science was like before the invention of the calculus. Calculations were more laborious and less precise; the realms which could be investigated were quite limited, given the lack of language to work in. Nothing too fast, nothing too small or large, nothing that changed too



much could be considered. Basically, everything which was beyond simple sense experience was inaccessible. It's amazing to think that all of this was unlocked with something that in no way is logical, intuitive, or practical, according to common sense. The infinitesimal is a nonsensical tool.

Between the Infinitesimal and the Infinite

Bernhard Riemann provides a bridge in science, between Leibniz' early phases in dealing with physics and modern day relativity, by allowing the imagination to be free from the chains of thinking of our world in terms of Euclidean solid bodies, as presented to our senses. During a time when the prominence of electro-magnetism (an invisible and powerful phenomenon) was challenging the dominance of Newtonian physics, Riemann pushed the limits of what people were comfortable thinking about.

Riemann expanded Gauss' work on anti-Euclidean curved surfaces to include an increasing number of dimensions, space being a particular case of a triply-extended magnitude. "It then follows as a necessary consequence that the propositions of geometry cannot be derived from general notions of magnitude, but that the properties which distinguish space from other conceivable triply-extended magnitudes are only to be deduced from experience... These matters of fact are—like all matters of fact—not necessary, but only of empirical certainty; they are hypotheses. We may therefore investigate their probability, which within the limits of observation is of course very great, and inquire about the justice of their extension beyond the limits of observation, on the side both of the infinitely great and of the infinitely small." This generalization freed geometry from the axioms of Euclid and allowed for the possibility of spherical and elliptical space—that is, a finite space whose nature is determined by natural forces. Just like Leibniz's, Riemann's achievements were wholly guided by his philosophical ambitions, as expressed in his philosophical fragments. Let's take a look at how Riemann deals with thinking of the non-visualizable.

From Riemann's Habilitation Dissertation: *III. Application to Space*.[6]

> "§ 3. It is upon the exactness with which we follow phenomena into the infinitely small that our knowledge of their causal relations essentially depends. The progress of recent centuries in the knowledge of mechanics depends almost entirely on the exactness of the construction which has become possible through the invention of the infinitesimal calculus... Now it seems that the empirical notions on which the metrical determinations of space are founded, the notion of a

6. See Riemann's Habilitation Dissertation (http://larouchepac.com/node/12479). Also try exploring some works on Gauss' history and method (http://larouchepac.com/narrowpath).

solid body and of a ray of light, cease to be valid for the infinitely small. The question of the validity of the hypotheses of geometry in the infinitely small is bound up with the question of the ground of the metric relations of space. . . . Therefore, either the reality which underlies space must form a discrete manifold, or we must seek the ground of its metric relations outside it, in binding forces which act upon it. This leads us into the domain of another science, that of physics, into which the object of today's proceedings does not allow us to enter."

Bernhard Riemann

Riemann states that, in order to deal with non-visualizables, infinitesimals and the structure of space, we have to leave the realm of mathematics and enter the domain of physics. The visible realm of symbols and objects in a Euclidean space is abandoned for the more imaginatively challenging world of processes.

Einstein's attempt, with his Relativity Theory, in bringing physics closer to this reality, was the next necessary step of Riemann's work. Of course, the practical scientists of Einstein's day saw his "theory" as just that, an abstract thought that had no importance in the domain of experimental physics, something which was useful only in the subject of philosophy. History has proven this to be silly.

Visualizing 4-Dimensional Space-Time

One of the consequences of the relativity theory is that physical laws determine the shape of space, the geometry in which everything occurs. Physical principles do not bend an already existing flat space, but it creates the effect that we call curved space.

On the basis of the general theory of relativity, space, as opposed to "what fills space," has no separate existence. If we imagine the gravitational field to be removed, there does not remain a space, but absolutely nothing, and also no "topological space." This is the idea of space, not as an empty box, but as a field, and is exactly what Riemann called for in searching for the metric relations of space "in the binding forces which act upon it." The nature of Gravity is your geometry of space.

If we are to gain more power and knowledge over our space-time (e.g., in long-distance space travel), we, at the very least, need to be able to conceive of it, as Einstein anticipated in a speech given to the Prussian Academy of Science.

> "No man can visualize even three dimensions. I think in four dimensions, but only abstractly. The human mind can picture these dimensions no more than it can envisage electricity. Nevertheless they are no less real than electromagnetism."

Einstein uses a simple analogy to show how someone could begin to think of these things abstractly:

> "Imagine a scene in two-dimensional space, for instance, the painting of a man reclining on a bench. A tree stands besides the bench. Then imagine that the man walks from the bench to a rock on the other side of the tree. He cannot reach the rock except by walking either *in front of* or *behind* the tree. This is impossible in two dimensional space. He can reach the rock only by an excursion into the third dimension.
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> "Now imagine another man sitting on the bench. How did the other man get there? Since two bodies cannot occupy the same place at the same time, he can have gotten there only before or after the first man moved. In other words, he must have moved in time. Time is the fourth dimension."

That's pretty simple to follow, but add to that the fact that these dimensions aren't merely linear or flat, but that the four dimensions curve in on themselves, and then you're dealing with something that would boggle the average person's mind.

"Can we picture to ourselves a three-dimensional universe which is finite, yet unbounded?"

(Hint: It's not a warped sphere or plane. That's not 3-D. Maybe you're thinking of the space inside the sphere? But what does it mean for that to be curved?)

"The usual answer to this question is 'No,' but that is not the right answer. The answer should be 'Yes.' I want to show that, without any extraordinary difficulty, we can illustrate the theory of a finite universe by means of a mental image, to which, with some practice, we shall soon grow accustomed. A geometrical-physical theory as such is incapable of being directly pictured, being merely a system of concepts. But these concepts serve the purpose of bringing a multiplicity of real or imaginary sensory experiences into connection in the mind. To 'visualize' a theory, or bring it home to one's mind, therefore means to give a representation to that abundance of experiences for which the theory supplies the schematic arrangement. My only aim has been to show that the human faculty of visualization is by no means bound to capitulate to non-Euclidean geometry."


Nicolaus of Cusa

In Einstein's speech,[7] he carefully walks people through how they could map the processes of our universe, to determine whether or not it is an actually curved space-time which is not infinite in extent, but also not constrained by borders—like the same conditions we have in two-dimensional space on a sphere. Again, we are impressed by the fact that Einstein boldly approaches thoughts which have no direct sense expression, either in chasing a beam of light or working with the curvature (beyond two dimensions) of the entire universe.

This conceptualizing of a multi-dimensional, curved space-time, which is finite, yet unbounded, is exactly what's occurring in the composition and performance of a classical piece of music.[8] When music is used to conjure up visual objects, as in Gustav Holst's "The Planets" Suite or in "The Carnival of the Animals,"[9] it's not music classically composed at the level of a Mozart or a Beethoven (which is the closest expression of preconscious thought).

The positivist would pull his or her hair out over what we're attempting to do. "If it's not directly observable," they would cry, "then it has no meaning!" "And how dare you compare science to something as subjective as music." They separate the creative imagination of man from science, turning science into the job of robots that gather data received from our animal senses. Yet the empiricists were completely defeated by a non-Euclidean space-time, not to mention the discovery of electrons and the whole advent of atomic science.[10] As we've seen,

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7. Geometry and Experience (http://www.relativitycalculator.com/pdfs/einstein_geometry_and_experience_1921.pdf)

8. See LaRouche, Lyndon *That Which Underlies Motivic Thorough-Composition* (http://american_almanac.tripod.com/motivic.htm) *EIR*, Sept. 1, 1995

9. *Camille Saint-Saëns, I'm sure, would agree.*

10. Take this actual response from the positivist manifesto, *Wissenschaftliche Weltauffassung: Der Wiener Kreis* (*The Scientific Conception of the World: The Vienna Circle*):

"Neatness and clarity are striven for, and dark distances and unfathomable depths rejected. In science there are no 'depths'; there is surface everywhere. Here is an affinity with the Sophists, not with the Platonists; with the Epicureans, not with the Pythagoreans; with all those who stand for earthly being and the here and now. The scientific world-conception knows no *unsolvable riddle*. Clarification of the traditional philosophical problems leads us partly to unmask them as pseudo-problems, and partly to transform them into empirical problems and thereby subject them to the judgment of experimental science. The method of this clarification is that of *logical analysis*. . .The metaphysician and the theologian believe, thereby misunderstanding themselves, that their statements say something, or that they denote a state of affairs. Analysis, however, shows that these statements say nothing but merely express a certain mood and spirit. To express such feelings for life can be a significant task. But the proper medium for doing so is art, for instance lyric poetry or music."

– i.e., not in science. Funny enough, as an appendix to the manifesto, the Vienna Circle listed Albert Einstein as a *leading representative of the scientific world-conception*, along with Bertrand Russell and Ludwig Wittgenstein. Here's what Einstein has to say to that:

"I am not a positivist. Positivism states that what cannot be observed does not exist. This conception is scientifically indefensible, for it is impossible to make valid affirmations of what people 'can' or 'cannot' observe. One would have to say 'only what we observe exists,' which is obviously false."

—Interview with Alfred Stern in *The Contemporary Jewish Record,* June 1945.

Albert Einstein

superior power of our ideas over the sensuous. This type of practice in thinking frees us from the oppression of the physical and from narrow thinking.

In the same way, we shouldn't allow geometry (a useful science in its own realm) to dominate physics. Visual thinking, even as simple analogy, cannot be a crutch for, or boundary on, creative thinking. As with all sensual tools, it should be seen, when dealing with fundamental principle, as a negative with regard to that which we're actually looking at, a shadow and a limited case of a universal, and as only truly useful when left behind by a seemingly infinite leap.

The fact that we cannot rely upon sensual thinking for direct reality, shouldn't be seen as a disadvantage, or something crippling, but instead as a greater power that only mankind has. From these few cases of extraordinary men and ideas from throughout history, it's clear that the human thinking which is far from resembling the perceived reality of the senses, actually gets us closer to the reality of what matters the most: the principles acting upon what we see. Why would going further into the recesses of our imagination, further away from what outside "reality" looks like, get us closer to the inner secrets of the universe? For one thing, the essence of the universe is not material objects. But also, the rigorous, distilled process of the creative mind is a principle in that universe. It is a mirror of the unseen, creative process of the "objective world." We still have to be in dialogue with the universe through means of the senses, by conducting physical experiments and reading data—we can't simply talk to ourselves and expect to find all the answers—but the non-verbal, non-visual imagination has to be explicitly nourished and tapped into, to keep that dialogue progressing. The first step is recognizing it, the creative principle of the human identity, as such. So let us continue to practice thinking in forms above the sights and sounds of everyday experience!

some of the most ground-breaking thoughts in science lie outside the reach of what can be directly visualized. Think back to Einstein's thought-experiment of chasing a beam of light, which led to his Theory of Special Relativity; or think about his ability to conceptualize the cosmology of our world. Whose advice would you take? Einstein's, or someone who likes to stare at things?

The Infinitesimal, Yet Infinite Mind

Cusa's work, and that of others, demonstrates that the human imagination, when contemplating the truly profound things of our world, is inspired by the Maximum (the infinite, or that which is undetectable to the senses), and is able, through learned ignorance, to embrace the infinite as a whole in the intellect, rather than being enslaved to it. Instead of being intimidated by the objects that overwhelm our senses (the incredible heights of mountains, the endless distance of the horizon of the ocean or even the beautiful expanse of the night time sky), instead of being reminded of the limits of our imagination, we go to these things on purpose to willfully summon up the image of the sensuous infinite, to exercise the